Results on a Direct Digital Receiver Operated with Fast Learning Networks

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Abstract — **In this paper learning algorithms are used to decode QPSK modulated signals in a direct conversion microwave/millimetre wave receiver using an application specific six port module. Two different algorithms, *K*-Means and Online Bayesian Network, are considered for operation of decoder to recover IQ data from modulated signals. Bit Error Rate (BER) results vs. Noise level (Eb/No) are presented including the case where the Local Oscillator (LO) is not locked to carrier signal.**

I. INTRODUCTION

In comparison to heterodyne receivers, direct conversion receivers present a number of advantages such as reducing circuit complexity and allowing a higher level of integration [1-4]. This paper presents simulation results (using Matlab) on a Six-Port Direct Receiver (SPDR). Its architecture is shown in Fig. 1. Learning algorithms are useful for this typê of receiver because of their online updating capacity, avoiding physical calibration procedures, in favor of on-line calibration.

The two inputs of the Six-Port (V_{RF} for the QPSK signal and V_{LO} for the Local Oscillator signal) are given by following equations :

$$V_{RF}(t) = \mathrm{Re}\left(\frac{A_{RF}}{\sqrt{2}}\left(I(t) + jQ(t)\right)\exp\left(j\Phi_{RF}(t)\right)\right) \quad (1)$$

and

$$V_{LO}(t) = \mathrm{Re}\left(A_{LO}\exp\left(j\Phi_{LO}(t)\right)\right) \quad (2)$$

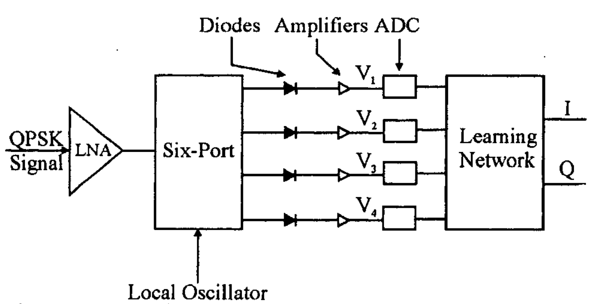

Fig. 1. Block diagram of the Six-Port Receiver with Learning

where

$$\Phi_{RF} = \omega_{RF}t + \varphi_{RF} \quad (3)$$

$$\Phi_{LO} = \omega_{LO}t + \varphi_{LO} \quad (4)$$

When the LO frequency is the same as the RF carrier frequency (ω_{LO}=ω_{RF}), A_{RF} = A_{LO} = A and φ_R = φ_S, the output equations are :

$$\begin{aligned} V_1 &= \lambda\frac{A^2}{2}\left(2-\left(-I(t)-Q(t)\right)\right) \\ V_2 &= \lambda\frac{A^2}{2}\left(2-\left(+I(t)-Q(t)\right)\right) \\ V_3 &= \lambda\frac{A^2}{2}\left(2-\left(+I(t)+Q(t)\right)\right) \\ V_4 &= \lambda\frac{A^2}{2}\left(2-\left(-I(t)+Q(t)\right)\right) \end{aligned} \quad (6)$$

Therefore, for a given QPSK symbol, , one of four outputs has a power level equal to $2\lambda A^2$, two other output ports have a power level equal to λA^2 and the remaining port has no power output.

Thus, the most two important problems are :

- noise which can prevent $A_S = A_R$
- the difference between the carrier frequency (ω_{RF}) and the LO frequency (ω_{LO}).

These important difficulties are solved by the Learning Network.

After an Analog to Digital Conversion of the four outputs of the Six-Port, the Learning Network transforms these output signals of the Six-Port into points in the IQ diagram. The I coordinate is a function of V_1 and V_2 and the Q coordinate is function of V_2 and V_3. Fig. 2 shows the IQ diagram for QPSK signals with a signal to noise ratio of 6dB and the LO and the RF-carrier signals have the same frequency. Fig. 3 shows the same case but with a difference in frequency between the LO signal and the carrier signal. The four clusters (for the four modulation states of the QPSK modulation) can be seen on these figures. To obtain these clusters, a collection of data samples must first be done. To work with clusters instead

First-In First-Out data base of samples in the learning algorithms.

II. Learning Algorithms

A. K-Means Algorithm

Considering only the IQ diagram, it is interesting to symbolize the four clusters by four models. The easiest possible model is the means of each clusters, and the corresponding algorithm is the *K*-Means algorithm [5]. To start, the following groups are built :

$$\mathbf{I}_k = \left\{\mathbf{X}_i, k = \arg\min_{j\in\{1,\dots,K\}}\left(\left\|\mathbf{M}_j - \mathbf{X}_i\right\|\right) i \in \{1,\dots,N\}\right\} \quad (7)$$

$\mathbf{X}_i$ is the i^{th} sample and $\mathbf{M}_j$ represents the j^{th} model. Because of the number of clusters, at least four models are needed so we use the 4-Means algorithm. The initial positions of the models are random positions.

The next step is to compute the new coordinates of the models using the following equation :

$$\mathbf{M}_k = \frac{\sum_{i=1}^{n_k} \mathbf{X}_{i,k}}{n_k} \quad (8)$$

where n_k is the number of elements of $\mathbf{I}_k$ and $\mathbf{X}_{i,k}$ represents an element of $\mathbf{I}_k$.

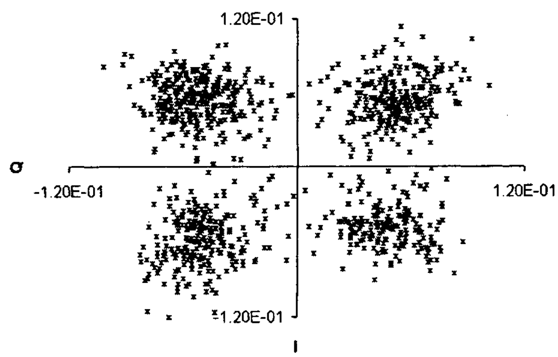

Fig. 2. IQ Diagram with signal to noise ratio (Eb/No) = 6dB

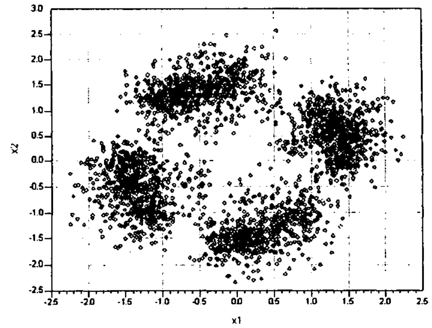

Fig. 3. IQ diagram with signal to noise ratio (Eb/No) = 6dB and a frequency difference (347 Hz) between the LO and the RF signals

Every time a new sample is received, an update of the collection is done and the coordinates of the models must be computed again using following equations :

$$\begin{aligned} &\frac{n_{k(new)}\mathbf{M}_{k(new)} + \mathbf{X}_{new}}{n_{k(new)} + 1} \rightarrow \mathbf{M}_{k(new)} \\ &\frac{n_{k(old)}\mathbf{M}_{k(old)} - \mathbf{X}_{old}}{n_{k(old)} - 1} \rightarrow \mathbf{M}_{k(old)} \\ &n_{k(new)} + 1 \rightarrow n_{k(new)} \\ &n_{k(old)} - 1 \rightarrow n_{k(old)} \end{aligned} \quad (9)$$

where $\mathbf{X}_{new}$ is the new sample
and $\mathbf{X}_{old}$ is the oldest in the data base.
$\mathbf{M}_{k(new)}$ is the model which is the nearest to $\mathbf{X}_{new}$
and $\mathbf{M}_{k(old)}$ is the nearest to $\mathbf{X}_{old}$.
$n_{k(old)}$ is the number of points associated to $\mathbf{M}_{k(old)}$
and $n_{k(new)}$ is the number of points associated to $\mathbf{M}_{k(new)}$.

The advantage of this iterative method is to allow a difference of frequency between the LO and the RF carrier frequency. Because the models are updated at every sample, they will follow in real time the evolution of the clusters and, in the case of a difference of frequency between the LO and the RF signals, they will rotate with the clusters. The major advantage of this solution is that the computation time is very short.

B. Fast Bayesian Algorithm

However, these models are relatively simple and some information can be lost. It is best to symbolize the clusters by the following models :

$$\mathbf{M}(i,t) = \{\boldsymbol{\mu}(i,t), \boldsymbol{\Sigma}(i,t), P(\mathbf{M}(i,t))\} \quad (10)$$

where at an instant t, $\mu(i,t)$ and $\Sigma(i,t)$ are the mean and the covariance matrix and $P(\mathbf{M}(i,t))$ the probability of the i^{th} model.

Using the Bayesian Networks Theory [6], the objective is to evaluate :

$$P(\mathbf{M}(i,t),\mathbf{X}(t)) = P(\mathbf{M}(i,t)|\mathbf{M}(i,t-1))P(\mathbf{X}(t)|\mathbf{M}(i,t)) \quad (11)$$

With the Bayes Law, we obtained :

$$P(\mathbf{X}(t)|\mathbf{M}(i,t)) = \frac{P(\mathbf{M}(i,t)|\mathbf{X}(t))P(\mathbf{X}(t))}{P(\mathbf{M}(i,t))} \quad (12)$$

The expressions $P(\mathbf{M}(i,t)|\mathbf{X}(t))$ and $P(\mathbf{M}(i,t))$ are computed with the following online Expectation-Maximization Algorithm [7] :

$$\mu(i,t+1) = \mu(i,t) - \frac{P(\mathbf{M}(i,t)|\mathbf{X}(t-N+1))}{\sum_{l=-1}^{N-2} P(\mathbf{M}(i,t)|\mathbf{X}(t-l))}(\mathbf{X}(t-N+1)-\mu(i,t)) + \frac{P(\mathbf{M}(i,t)|\mathbf{X}(t+1))}{\sum_{l=-1}^{N-2} P(\mathbf{M}(i,t)|\mathbf{X}(t-l))}(\mathbf{X}(t+1)-\mu(i,t)) \quad (13)$$

$$\Sigma(i,t+1) = \Sigma(i,t) - \frac{P(\mathbf{M}(i,t)|\mathbf{X}(t-N+1))}{\sum_{l=-1}^{N-2} P(\mathbf{M}(i,t)|\mathbf{X}(t-l))}\gamma(\mathbf{X}(t-N+1)-\mu(i,t)) + \frac{P(\mathbf{M}(i,t)|\mathbf{X}(t-N+1))}{\sum_{l=-1}^{N-2} P(\mathbf{M}(i,t)|\mathbf{X}(t-l))}\Sigma(i,t) + \frac{P(\mathbf{M}(i,t)|\mathbf{X}(t+1))}{\sum_{l=-1}^{N-2} P(\mathbf{M}(i,t)|\mathbf{X}(t-l))}(\gamma(\mathbf{X}(t+1)-\mu(i,t))-\Sigma(i,t)) \quad (14)$$

with

$$\gamma(x) = xx^T \quad (15)$$

$$P(\mathbf{M}(i,t)) \leftarrow \frac{1}{N}\sum_{l=0}^{N-1}\frac{P(\mathbf{M}(i,t))P(\mathbf{X}(t-l)|\mathbf{M}(i,t))}{\sum_{j=1}^{4} P(\mathbf{M}(j,t))P(\mathbf{X}(t-l)|\mathbf{M}(j,t))} \quad (16)$$

If $\Delta(i,t+1) = \sum_{l=-1}^{N-2} P(\mathbf{M}(i,t)|\mathbf{X}(t-l))$, we can write :

$$\Delta(i,t+1) \approx \left(1-\frac{1}{N}\right)\Delta(i,t) + P(\mathbf{M}(i,t)|\mathbf{X}(t+1)) \quad (17)$$

So we can evaluate :

$$P(\mathbf{M}(i,t)|\mathbf{X}(t)) = \frac{\frac{1}{2\pi|\Sigma(i,t)|^{0.5}}\exp(\alpha(\mu(i,t),\Sigma(i,t),\mathbf{X}(t)))}{\sum_{j=1}^{4}\frac{1}{2\pi|\Sigma(j,t)|^{0.5}}\exp(\alpha(\mu(j,t),\Sigma(j,t),\mathbf{X}(t)))} \quad (18)$$

with

$$\alpha(\mu(i,t),\Sigma(i,t),\mathbf{X}(t)) = -\frac{1}{2}(\mathbf{X}(t)-\mu(i,t))^T\Sigma(i,t)^{-1}(\mathbf{X}(t)-\mu(i,t)) \quad (19)$$

We also have :

$$P(\mathbf{X}(t)) = \sum_{j=1}^{4} P(\mathbf{M}(j,t))P(\mathbf{X}(t)|\mathbf{M}(j,t)) \quad (20)$$

where $P(\mathbf{M}(i,t)|\mathbf{M}(i,t-1)) = 1-3/4s$ if the new sample and the last sample belong to the same model and $P(\mathbf{M}(i,t)|\mathbf{M}(i,t-1)) = 3/4s$ if the new sample and the last sample do not belong to the same model. s is the number of sample for a symbol.

The new sample belongs to the model which maximizes $P(\mathbf{M}(i,t),\mathbf{X}(t))$.

The above algorithm has the same advantages than the *K*-Means algorithm (i.e. short computation time and iterative/online update of the models). But this algorithm is more robust than the K-Means algorithm so we can expect an improved performance.

III. Performance of Algorithms

A. Bit Error Rate

To evaluate the performances of the two algorithms, the Bit Error Rate (BER) vs. the signal to noise ratio (E_b/N_0) was calculated. However, the size of the sample data base is important and the simulations show the optimal size for the K-Means algorithm is 240 samples and 300 for the Bayesian algorithm. More samples are used by the Bayesian algorithm because more information is needed to evaluate the models.

The results are presented on Fig. 4. The Bayesian algorithm seems to be the best solution. The uncertainty on the noise level explains why the BER of the Bayesian Algorithm is sometimes "under" the BER of the ideal QPSK receiver.

For E_b/N_0 higher than 5dB, Matlab was not able to handle data needed to obtain a good estimation of the BER, and higher values of signal to noise ratio were not calculated.

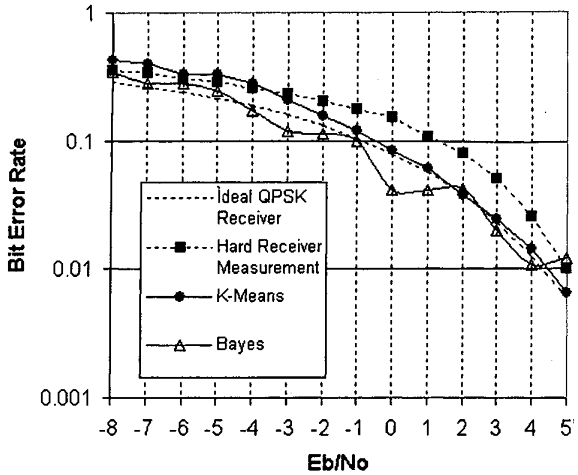

Fig. 4. BER vs. Signal to Noise Ratio (Eb/No)

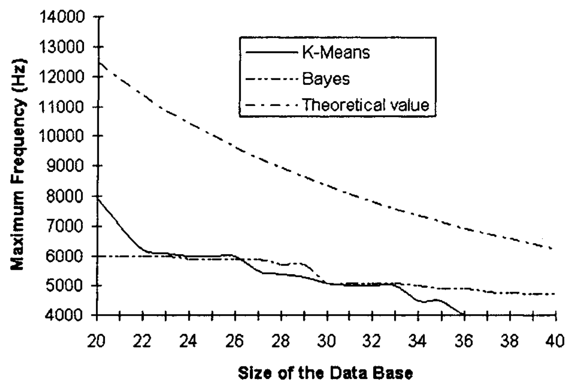

Fig. 5. Maximum difference frequency between RF carrier and LO vs. size of date base

B. Maximum Difference of Frequency

The two algorithms allow for a difference of frequency between the LO and the RF signals. We evaluate the maximum of this difference by taking into account the size of the data base.

Fig. 5 shows the theoretical maximum and the maximum for Bayesian and *K*-Means algorithms. The simulations were done for a carrier frequency of 26.5GHz, a data rate of 1Mbit/s and a sampling frequency of 10MHz for the ADC of Fig. 1. For the two learning algorithms, as the number of symbols increases, the maximum difference frequency decreases. Indeed many symbols used in the learning period imply a long collecting period and modifications of the IQ diagram. Modifications of the IQ diagram are less important if the frequency difference is smaller. The Bayesian Network is less dependant on the number of symbols. If the highest frequency difference is needed, *K*-Means Network is the best solution.

IV. CONCLUSION

Learning networks provide useful methods to operate direct digital six port unimode (QPSK) receivers in presence of relatively high noise levels ($E_b/N_0 > -4$dB). Learning Network can operate a SPDR with a frequency difference between LO and RF carrier.

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